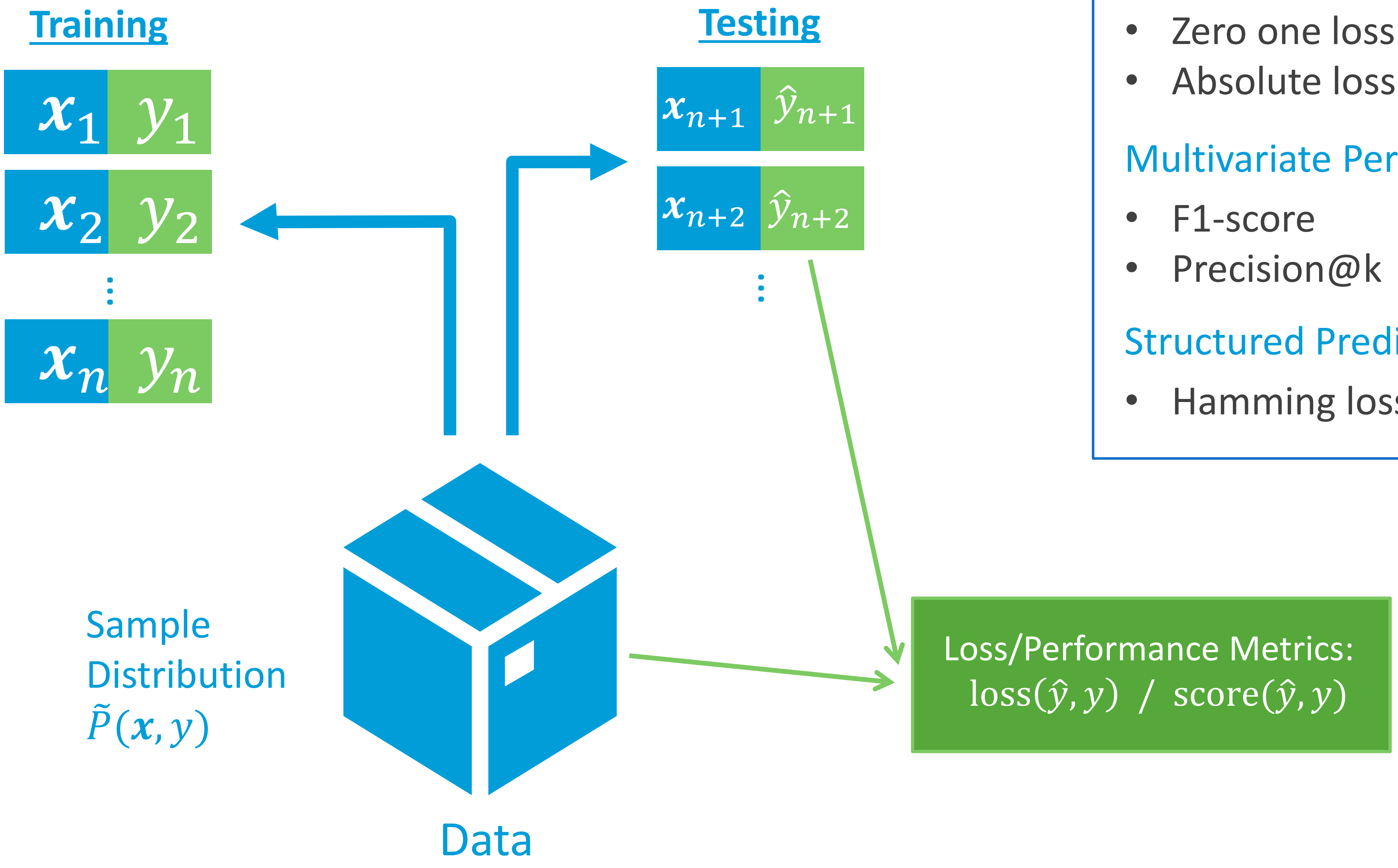
Performance-Aligned Learning Algorithms
with Statistical Guarantees

RIZAL FATHONY

UIC COMPUTER SCIENCE

Supervised Learning | Classification



Multiclass Classification

- Zero one loss / accuracy metric
- Absolute loss (for ordinal regression)

Multivariate Performance

- F1-score
- Precision@k

Structured Prediction

- Hamming loss

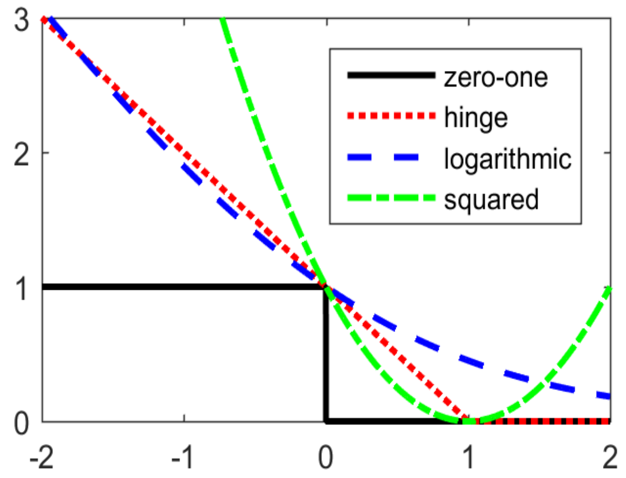
Outline

Motivation

Empirical Risk Minimization (ERM)

- Assume a family of parametric hypothesis function f (e.g. linear discriminator)
- Find the hypothesis f^* that minimize the empirical risk:

$$\min_f \frac{1}{n} \sum_{i=1}^{n} \text{loss}(f(\mathbf{x}_i), y_i) = \min_f \mathbb{E}_{\mathbf{X}, Y \sim \tilde{P}} \left[\text{loss}(f(\mathbf{X}), Y) \right]$$



Intractable optimization, non-convex, non-continuous

Convex surrogate loss need to be employed!

A desirable property of convex surrogates:

Fisher Consistency

Under ideal condition: optimize surrogate → minimizes the loss metric

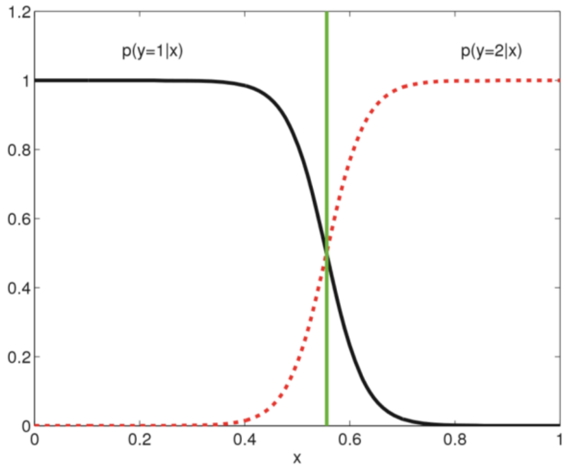
(given the true distribution and fully expressive model)

Two Main Approaches

1 Probabilistic Approach

- Construct prediction probability model
- Employ the logistic loss surrogate

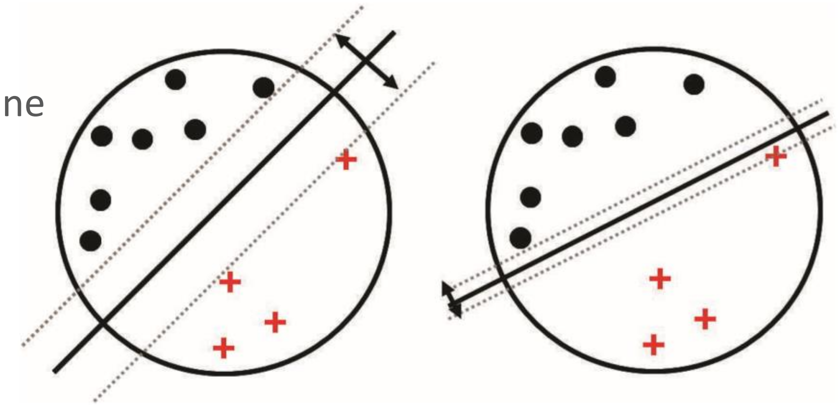
Logistic Regression, Conditional Random Fields (CRF)



2 Maximum Margin Approach

- Maximize the margin that separates correct prediction from the incorrect one
- Employ the hinge loss surrogate

Support Vector Machine (SVM), Structured SVM



* Pictures are taken from MLPP book (Kevin Murphy)

Multiclass Classification | Logistic Regression vs SVM

1 Multiclass Logistic Regression

✓ Statistical guarantee of Fisher consistency
(minimizes the zero-one loss metric in the limit)

✗ No dual parameter sparsity

2 Multiclass SVM

✓ Computational efficiency
(via the kernel trick & dual parameter sparsity)

✗ Current multiclass SVM formulations:

- Lack Fisher consistency property, or
- Doesn't perform well in practice

Structured Prediction| CRF vs Structured SVM

1 Conditional Random Fields (CRF)	2 Structured SVM
✓ Statistical guarantee of Fisher consistency	✗ No Fisher consistency guarantee
✗ No easy mechanism to incorporate customized loss/performance metrics	✓ Flexibility to incorporate customized loss/performance metrics
✗ Computation of the normalization term may be intractable	✓ Relatively more efficient in computation

New Learning Algorithms?

- ✓ Align better with the loss/performance metric
  (by incorporating the metric into its learning objective)
- ✓ Provide Fisher consistency guarantee
- ✓ Computationally efficient
- ✓ Perform well in practice

How?

Robust adversarial learning approach

"What predictor best maximizes the performance metric (or minimizes the loss metric) in the worst case given the statistical summaries of the empirical distributions?"

Robust Adversarial Formulation

Robust Adversarial Formulation (Asif et.al, 2015; Grunwald & Dawid, 2004; Topsoe, 1979)

Original Loss Metric

Non-convex, non-continuous

$$\min_f \mathbb{E}_{\mathbf{X},Y\sim\tilde{P}}\left[\text{loss}(f(\mathbf{X}),Y)\right]$$

Approximate the loss with convex surrogates

Empirical Risk Minimization

$$\min_f \mathbb{E}_{\mathbf{X},Y\sim\tilde{P}}\left[\text{surrogate}(f(\mathbf{X}),Y)\right]$$

$$\hat{y} = \operatorname*{argmax}_j f_j(\mathbf{x})$$

Probabilistic prediction $\hat{P}(\hat{Y}|\mathbf{X})$

$$\min_{\hat{P}(\hat{Y}|\mathbf{X})} \mathbb{E}_{\mathbf{X},Y\sim\tilde{P};\hat{Y}|\mathbf{X}\sim\hat{P}}\left[\text{loss}(\hat{Y},Y)\right]$$

Evaluate against an adversary, instead of using empirical data

Adversary's probabilistic prediction $\check{P}(\check{Y}|\mathbf{X})$

$$\min_{\hat{P}(\hat{Y}|\mathbf{X})} \max_{\check{P}(\check{Y}|\mathbf{X})} \mathbb{E}_{\mathbf{X},Y\sim\tilde{P};\hat{Y}|\mathbf{X}\sim\hat{P};\check{Y}|\mathbf{X}\sim\check{P}}\left[\text{loss}(\hat{Y},\check{Y})\right]$$

Constraint the **statistics** of the **adversary**'s distribution to match the **empirical** statistics

Robust Adversarial Formulation

$$\min_{\hat{P}(\hat{Y}|\mathbf{X})} \max_{\check{P}(\check{Y}|\mathbf{X})} \mathbb{E}_{\mathbf{X},Y\sim\tilde{P};\hat{Y}|\mathbf{X}\sim\hat{P};\check{Y}|\mathbf{X}\sim\check{P}}\left[\text{loss}(\hat{Y},\check{Y})\right]$$

$$\text{s.t. } \mathbb{E}_{\mathbf{X}\sim\tilde{P};\check{Y}|\mathbf{X}\sim\check{P}}[\phi(\mathbf{X},\check{Y})] = \mathbb{E}_{\mathbf{X},Y\sim\tilde{P}}[\phi(\mathbf{X},Y)]$$

Robust Adversarial Dual Formulation

Primal:

$$\min_{\hat{P}(\hat{Y}|\mathbf{X})} \max_{\check{P}(\check{Y}|\mathbf{X})} \mathbb{E}_{\mathbf{X}\sim\tilde{P};\hat{Y}|\mathbf{X}\sim\hat{P};\check{Y}|\mathbf{X}\sim\check{P}} \left[\text{loss}(\hat{Y},\check{Y})\right]$$

$$\text{subject to: } \mathbb{E}_{\mathbf{X}\sim\tilde{P};\check{Y}|\mathbf{X}\sim\check{P}}[\phi(\mathbf{X},\check{Y})] = \mathbb{E}_{\mathbf{X},Y\sim\tilde{P}}[\phi(\mathbf{X},Y)]$$

Lagrange multiplier, minimax duality

Dual:

$$\min_{\theta} \mathbb{E}_{\mathbf{X},Y\sim\tilde{P}} \underbrace{\max_{\check{P}(\check{Y}|\mathbf{X})} \min_{\hat{P}(\hat{Y}|\mathbf{X})} \mathbb{E}_{\hat{Y}|\mathbf{X}\sim\hat{P};\check{Y}|\mathbf{X}\sim\check{P}} \left[\text{loss}(\hat{Y},\check{Y}) + \theta^\intercal \left(\phi(\mathbf{X},\check{Y}) - \phi(\mathbf{X},Y)\right)\right]}$$

ERM with the adversarial surrogate loss (AL):

$$AL(\mathbf{x},y,\theta) = \max_{\check{P}(\check{Y}|\mathbf{x})} \min_{\hat{P}(\hat{Y}|\mathbf{x})} \mathbb{E}_{\hat{Y}|\mathbf{x}\sim\hat{P};\check{Y}|\mathbf{x}\sim\check{P}} \left[\text{loss}(\hat{Y},\check{Y}) + \theta^\intercal \left(\phi(\mathbf{x},\check{Y}) - \phi(\mathbf{x},y)\right)\right]$$

Convex in θ

Simplified notation

$$AL(\mathbf{f},y) = \max_{\mathbf{q}\in\Delta} \min_{\mathbf{p}\in\Delta} \mathbf{p}^\intercal \mathbf{L} \mathbf{q} + \mathbf{f}^\intercal \mathbf{q} - f_y$$

where:

$$p_i = \hat{P}(\hat{Y} = i|\mathbf{x})$$

$$q_i = \check{P}(\check{Y} = i|\mathbf{x})$$

$$f_i = \theta^\intercal \phi(\mathbf{x}, i)$$

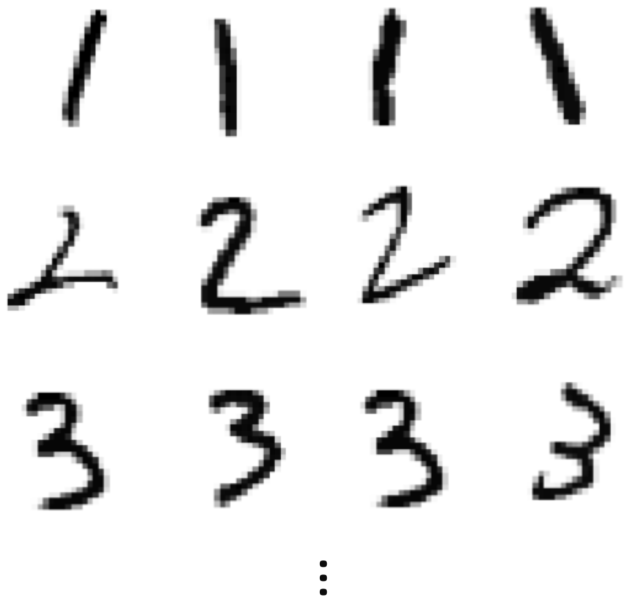
Multiclass Zero-One Classification

Based on:

Rizal Fathony, Anqi Liu, Kaiser Asif, Brian D. Ziebart. "*Adversarial Multiclass Classification: A Risk Minimization Perspective*". Advances in Neural Information Processing Systems 29 (NIPS), 2016.

Multiclass Classification | Zero-One Loss

Example: Digit Recognition



1

2

3

⋮

Loss Metric: Zero-One Loss

Loss Metric:
$\text{loss}(\hat{y}, y) = I(\hat{y} \neq y)$

$$\mathbf{L} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Multiclass Classification | Related Works

Multiclass Support Vector Machine (SVM)

	Fisher Consistent? (Tewari and Bartlett, 2007) (Liu, 2007)	**Perform well** in low dimensional feature? (Dogan et.al., 2016)
1. The WW Model (Weston et.al., 2002)	✗	✓
2. The CS Model (Crammer and Singer, 1999)	✗	✓
3. The LLW Model (Lee et.al., 2004)	✓	✗

1. The WW Model (Weston et.al., 2002)

$$\text{loss}_{\text{WW}}(\mathbf{x}_i, y_i) = \sum_{j \neq y_i} [1 - (f_{y_i}(\mathbf{x}_i) - f_j(\mathbf{x}_i))]_+$$

Relative Margin Model

2. The CS Model (Crammer and Singer, 1999)

$$\text{loss}_{\text{CS}}(\mathbf{x}_i, y_i) = \max_{j \neq y_i} [1 - (f_{y_i}(\mathbf{x}_i) - f_j(\mathbf{x}_i))]_+$$

Relative Margin Model

3. The LLW Model (Lee et.al., 2004)

$$\text{loss}_{\text{LLW}}(\mathbf{x}_i, y_i) = \sum_{j \neq y_i} [1 + f_j(\mathbf{x}_i)]_+$$

with: $\sum_j f_j(\mathbf{x}_i) = 0$

Absolute Margin Model

Adversarial Surrogate Loss for Zero-One Loss ($AL^{0\text{-}1}$)

Adversarial Surrogate Loss

$$AL(\mathbf{f}, y) = \max_{\mathbf{q}\in\Delta} \min_{\mathbf{p}\in\Delta} \mathbf{p}^\intercal \mathbf{L}\mathbf{q} + \mathbf{f}^\intercal \mathbf{q} - f_y$$

Convert to Linear Program

$$\begin{aligned} AL(\mathbf{f}, y) = \max_{\mathbf{q},v}\ & v + \mathbf{f}^\intercal \mathbf{q} - f_y \\ \text{s.t.: } & \mathbf{L}_{(i,:)}\mathbf{q} \geq v \quad \forall i \in [k] \\ & q_i \geq 0 \quad \forall i \in [k] \\ & \mathbf{q}^\intercal \mathbf{1} = 1 \end{aligned}$$

Convex Polytope formed by the constraints

$$\mathbb{C} = \left\{ \begin{bmatrix} \mathbf{q} \\ v \end{bmatrix} \middle| \mathbf{A} \begin{bmatrix} \mathbf{q} \\ v \end{bmatrix} \geq \mathbf{b}, \quad \text{where} \quad \mathbf{A} = \begin{bmatrix} \mathbf{L} & -\mathbf{1} \\ \mathbf{I} & \mathbf{0} \\ \mathbf{1}^\intercal & 0 \\ -\mathbf{1}^\intercal & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ 1 \\ -1 \end{bmatrix} \right\}$$

Example for a four class classification

$$\begin{array}{c} \text{1st block} \\ \\ \text{2nd block} \\ \\ \text{3rd block} \end{array} \begin{bmatrix} 0 & 1 & 1 & 1 & -1 \\ 1 & 0 & 1 & 1 & -1 \\ 1 & 1 & 0 & 1 & -1 \\ 1 & 1 & 1 & 0 & -1 \\ \hdashline 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \hdashline 1 & 1 & 1 & 1 & 0 \\ -1 & -1 & -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ v \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

Extreme points of the (bounded) polytope

There is always an optimal solution that is an extreme point of the domain.

Computing AL =
finding the best extreme point

$AL^{0\text{-}1}$ | Convex Polytope

Convex Polytope of the $AL^{0\text{-}1}$

$$\mathbb{C} = \left\{ \begin{bmatrix} \mathbf{q} \\ v \end{bmatrix} \middle| \mathbf{A} \begin{bmatrix} \mathbf{q} \\ v \end{bmatrix} \geq \mathbf{b}, \quad \text{where} \quad \mathbf{A} = \begin{bmatrix} \mathbf{L} & -\mathbf{1} \\ \mathbf{I} & \mathbf{0} \\ \mathbf{1}^\intercal & 0 \\ -\mathbf{1}^\intercal & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ 1 \\ -1 \end{bmatrix} \right\}$$

$$\mathbf{L} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Extreme points of the polytope

$$D = \left\{ \begin{bmatrix} \mathbf{q} \\ v \end{bmatrix} = \frac{1}{|S|} \begin{bmatrix} \sum_{i \in S} \mathbf{e}_i \\ |S| - 1 \end{bmatrix} \middle| \emptyset \neq S \subseteq [k] \right\}$$

$\boldsymbol{e}_i$ is a vector with a single 1 at the *i*-th index, and 0 elsewhere.

$$[k] \triangleq \{1, \ldots, k\}$$

The Adversarial Surrogate Loss for Zero-One Loss Metrics ($AL^{0\text{-}1}$)

$$AL^{0\text{-}1}(\mathbf{f}, y) = \max_{S \subseteq [k],\, S \neq \emptyset} \frac{\sum_{i \in S} f_i + |S| - 1}{|S|} - f_y$$

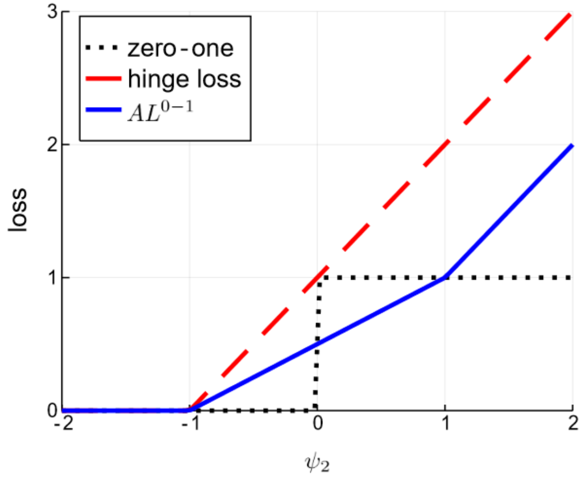
Computation of $AL^{0\text{-}1}$

- Sort f_i in non-increasing order
- Incrementally add potentials to the set S, until adding more potential decrease the loss value

$O(k \log k)$, where k is the number of classes

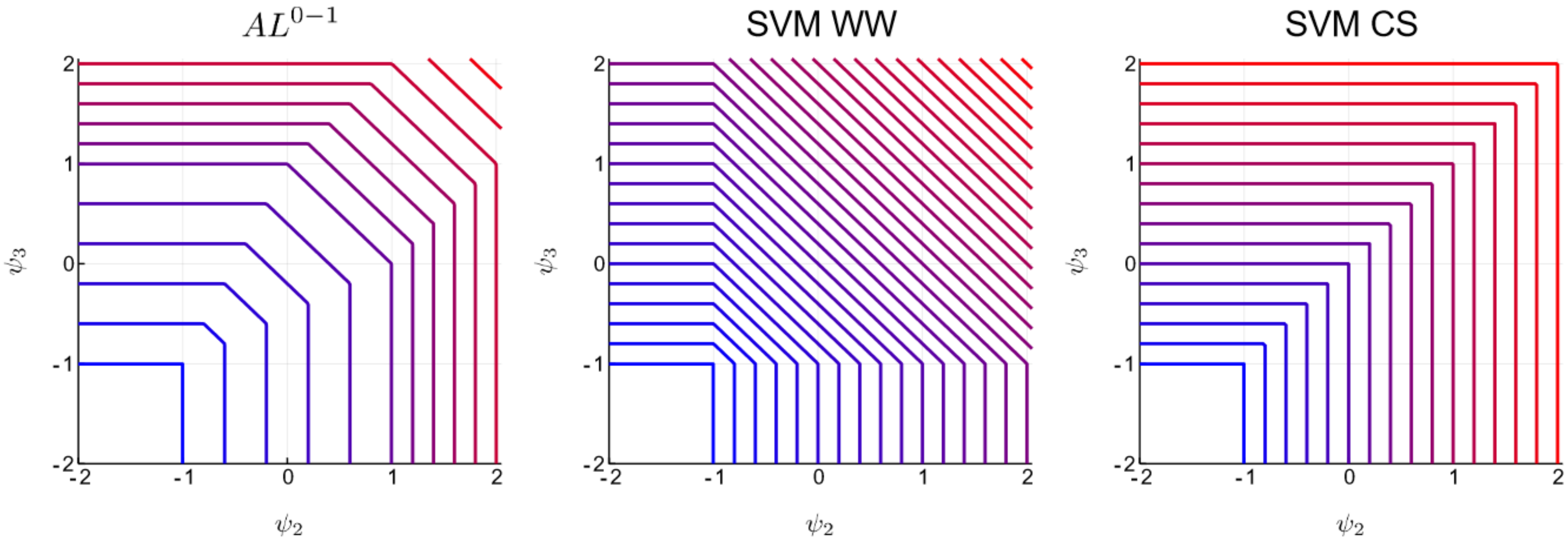
AL$^{0-1}$ | Loss Surface

Binary Classification



- Plots over the space of potential differences $\psi_i = f_i - f_y$
- The true label is $y = 1$

Three Class Classification



Fisher Consistency

Fisher Consistency Requirement in Classification

$$f^* \in \mathcal{F}^* \triangleq \operatorname*{argmin}_f \mathbb{E}_{Y|\mathbf{x}\sim P}\left[\text{surrogate}_f(\mathbf{x}, Y)\right] \;\Rightarrow\; \operatorname*{argmax}_y f^*(\mathbf{x}, y) \subseteq \mathcal{Y}^\diamond \triangleq \underbrace{\operatorname*{argmin}_{y'} \mathbb{E}_{Y|\mathbf{x}\sim P}[\text{loss}(y', Y)]}_{\text{Bayes risk minimizer}}$$

- $P(Y|\boldsymbol{x})$ is the true conditional distribution
- f is optimized over all measurable functions

The property of the minimizer for AL

Loss reflective property of AL, for any loss metrics

$f^*(\mathbf{x}, y) + \text{loss}(y^\diamond, y)$ = constant, i.e., is invariant to y

$y^\diamond$ is the Bayes risk minimizer.

↓

$$\text{argmax}_y\, f^*(\mathbf{x}, y) = \text{argmin}_y\, \mathbf{L}_{(y^\diamond, y)}$$

↓

Fisher consistent

AL$^{0\text{-}1}$ | Optimization

Primal

Stochastic sub-gradient descent

$$\partial_\theta \, AL^{0\text{-}1}(\mathbf{x}, y, \theta) \ni \frac{1}{|S^*|} \sum_{j \in S^*} \phi(\mathbf{x}, j) - \phi(\mathbf{x}, y)$$

S^* is the set that maximize AL$^{0\text{-}1}$

Kernel trick

input space $\boldsymbol{x}_i$ → rich feature space $\omega(\boldsymbol{x}_i)$

Compute the dot products

$$K(\mathbf{x}_i, \mathbf{x}_j) = \omega(\mathbf{x}_i) \cdot \omega(\mathbf{x}_j)$$

Dual

Constrained Primal QP

$$\min_\theta \frac{1}{2}\|\theta\|^2 + C \sum_{i=1}^{n} \xi_i$$

subject to: $\xi_i \geq \Delta_{i,k} \quad \forall i \in \{1, \dots n\} k \in \{1, \dots, 2^{|\mathcal{Y}|} - 1\}$

Δ enumerate all $2^{|\mathcal{Y}|} - 1$ possible values of AL$^{0\text{-}1}$ for each sample

Dual QP Formulation

$$\max_{\boldsymbol{\alpha}} \sum_{i=1}^{n} \sum_{k=1}^{2^{|\mathcal{Y}|}-1} \nu_{i,k} \, \alpha_{i,k} - \frac{1}{2} \sum_{i,j=1}^{m} \sum_{k,l=1}^{2^{|\mathcal{Y}|}-1} \alpha_{i,k}\alpha_{j,l} \, [\Lambda_{i,k} \cdot \Lambda_{j,l}]$$

subject to $\alpha_{i,k} \geq 0, \; \sum_{k=1}^{2^{|\mathcal{Y}|}-1} \alpha_{i,k} = C, \; i \in \{1, \dots, n\}, \; k \in \{1, \dots, 2^{|\mathcal{Y}|} - 1\}$

where:

$\Lambda_{i,k} = \frac{d\Delta_{i,k}}{d\theta}$, and $\nu_{i,k}$ is the constant part of $\Delta_{i,k}$

$$\Lambda_{i,k} \cdot \Lambda_{j,l} = c_{(i,k),(j,l)} \, K(\mathbf{x}_i, \mathbf{x}_j)$$

for some constants $c_{(i,k)}$ and $c_{(j,l)}$

Dual Optimization

Exp. number of constraints (primal) → Exp. number of variables (dual)

Constraint Generation Algorithm

$AL^{0\text{-}1}$ | Experiments

Dataset properties and $AL^{0\text{-}1}$ constraints

Dataset		Properties				SVM	$AL^{0\text{-}1}$ constraints added and active			
		#class	#train	# test	#feat.	constraints	Linear kernel		Gauss. kernel	
(1)	iris	3	105	45	4	210	213	13	223	38
(2)	glass	6	149	65	9	745	578	125	490	252
(3)	redwine	10	1119	480	11	10071	5995	1681	3811	1783
(4)	ecoli	8	235	101	7	1645	614	117	821	130
(5)	vehicle	4	592	254	18	1776	1310	311	1201	248
(6)	segment	7	1617	693	19	9702	4410	244	4312	469
(7)	sat	7	4435	2000	36	26610	11721	1524	11860	6269
(8)	optdigits	10	3823	1797	64	34407	7932	597	10072	2315
(9)	pageblocks	5	3831	1642	10	15324	9459	427	9155	551
(10)	libras	15	252	108	90	3528	1592	389	1165	353
(11)	vertebral	3	217	93	6	434	344	78	342	86
(12)	breasttissue	6	74	32	9	370	258	65	271	145

AL^{0-1} | Experiments | Results

Results for Linear Kernel and Gaussian Kernel

The **mean (standard deviation)** of the **accuracy.** Bold numbers: **best** or **not significantly worse** than the best

D	Linear Kernel				Gaussian Kernel			
	AL^{0-1}	WW	CS	LLW	AL^{0-1}	WW	CS	LLW
(1)	**96.3** (3.1)	**96.0** (2.6)	**96.3** (2.4)	79.7 (5.5)	**96.7** (2.4)	**96.4** (2.4)	**96.2** (2.3)	95.4 (2.1)
(2)	**62.5** (6.0)	**62.2** (3.6)	**62.5** (3.9)	52.8 (4.6)	**69.5** (4.2)	66.8 (4.3)	**69.4** (4.8)	**69.2** (4.4)
(3)	**58.8** (2.0)	**59.1** (1.9)	56.6 (2.0)	57.7 (1.7)	63.3 (1.8)	64.2 (2.0)	64.2 (1.9)	**64.7** (2.1)
(4)	**86.2** (2.2)	85.7 (2.5)	**85.8** (2.3)	74.1 (3.3)	**86.0** (2.7)	84.9 (2.4)	**85.6** (2.4)	**86.0** (2.5)
(5)	**78.8** (2.2)	**78.8** (1.7)	**78.4** (2.3)	69.8 (3.7)	**84.3** (2.5)	**84.4** (2.6)	83.8 (2.3)	**84.4** (2.6)
(6)	94.9 (0.7)	94.9 (0.8)	**95.2** (0.8)	75.8 (1.5)	**96.5** (0.6)	**96.6** (0.5)	96.3 (0.6)	96.4 (0.5)
(7)	84.9 (0.7)	**85.4** (0.7)	84.7 (0.7)	74.9 (0.9)	**91.9** (0.5)	**92.0** (0.6)	**91.9** (0.5)	**91.9** (0.4)
(8)	**96.6** (0.6)	96.5 (0.7)	96.3 (0.6)	76.2 (2.2)	98.7 (0.4)	98.8 (0.4)	98.8 (0.3)	**98.9** (0.3)
(9)	96.0 (0.5)	96.1 (0.5)	**96.3** (0.5)	92.5 (0.8)	**96.8** (0.5)	96.6 (0.4)	96.7 (0.4)	96.6 (0.4)
(10)	**74.1** (3.3)	72.0 (3.8)	71.3 (4.3)	34.0 (6.4)	83.6 (3.8)	83.8 (3.4)	**85.0** (3.9)	83.2 (4.2)
(11)	**85.5** (2.9)	**85.9** (2.7)	**85.4** (3.3)	79.8 (5.6)	**86.0** (3.1)	**85.3** (2.9)	85.5 (3.3)	84.4 (2.7)
(12)	**64.4** (7.1)	59.7 (7.8)	**66.3** (6.9)	58.3 (8.1)	**68.4** (8.6)	**68.1** (6.5)	**66.6** (8.9)	**68.0** (7.2)
avg	81.59	81.02	81.25	68.80	85.14	84.82	85.00	84.93
#b	9	6	8	0	9	6	6	7

Multiclass Zero-One Classification

	Fisher Consistent?	**Perform well** in low dimensional feature?
1. The SVM WW Model (Weston et.al., 2002) Relative Margin Model	✗	✓
2. The SVM CS Model (Crammer and Singer, 1999) Relative Margin Model	✗	✓
3. The SVM LLW Model (Lee et.al., 2004) Absolute Margin Model	✓	✗
4. The $AL^{0\text{-}1}$ (Adversarial Surrogate Loss) Relative Margin Model	✓	✓

Other results

General Multiclass Classification

General Multiclass Classification

1. Zero-One Loss Metric (NIPS 2016)
2. Ordinal Classification with the Absolute Loss Metric (NIPS 2017)
3. Ordinal Classification with the Squared Loss Metric
4. Weighted Multiclass Loss Metrics
5. Classification with Abstention / Reject Option

(JMLR submission in preparation)

Conditional Graphical Models

Based on:

Rizal Fathony, Ashkan Rezaei, Mohammad Bashiri, Xinhua Zhang, Brian D. Ziebart. "*Distributionally Robust Graphical Models*". Advances in Neural Information Processing Systems 31 (NIPS), 2018

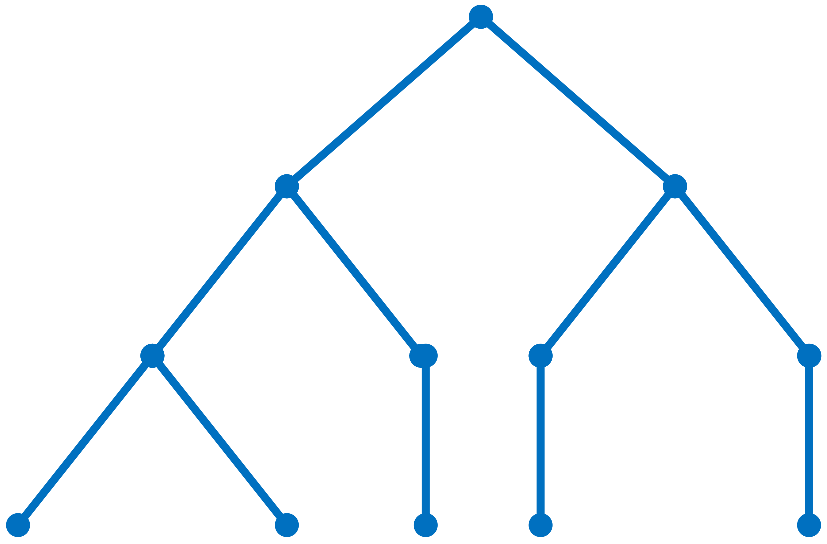
Conditional Graphical Models

Some Popular Graphical Structure in Structured Prediction

Chain Structure

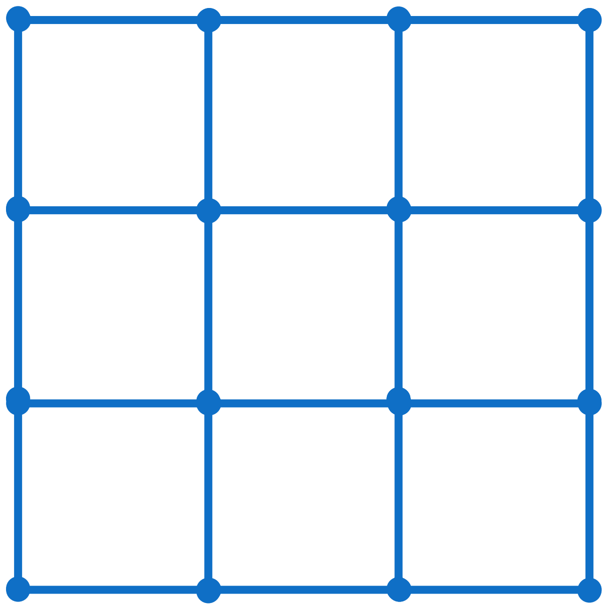
Activity Prediction, Sequence Tagging, NLP tasks: e.g. Named Entity Recognition

Tree Structure



Parse Tree-Based NLP tasks:
Semantic Role Labeling
and Sentiment Analysis

Lattice Structure



Computer Vision Tasks:
e.g. Image Segmentation

Previous Approaches for Conditional Graphical Models

1 Conditional Random Fields (CRF)

(Lafferty et. al., 2001)

✓ **Fisher Consistent**

Produce Bayes optimal prediction in ideal case.

✗ **No easy mechanism to incorporate customized loss/performance metrics**

The algorithm optimized the conditional likelihood.
Loss/performance metric-based prediction can be performed after learning process.

2 Structured SVM (SSVM)

(Tsochantaridis et. al., 2005)

✗ **No Fisher consistency guarantee**

Based on Multiclass SVM-CS.
Not consistent for distribution with no majority label.

✓ **Align with the loss/performance metrics**

The algorithm accept customized loss/performance metric in its optimization objective.

Adversarial Graphical Models (AGM)

Primal:

$$\min_{\hat{P}(\hat{\mathbf{y}}|\mathbf{x})} \max_{\check{P}(\check{\mathbf{y}}|\mathbf{x})} \mathbb{E}_{\mathbf{X}\sim\tilde{P};\hat{\mathbf{Y}}|\mathbf{X}\sim\hat{P};\check{\mathbf{Y}}|\mathbf{X}\sim\check{P}} \left[loss(\hat{\mathbf{Y}}, \check{\mathbf{Y}}) \right] \ s.t.: \ \mathbb{E}_{\mathbf{X}\sim\tilde{P};\check{\mathbf{Y}}|\mathbf{X}\sim\check{P}} \left[\Phi(\mathbf{X}, \check{\mathbf{Y}}) \right] = \tilde{\Phi}$$

- Feature function $\Phi(\mathbf{X}, \mathbf{Y})$ is additively decomposed over cliques, $\Phi(\mathbf{x}, \mathbf{y}) = \sum_c \phi(\mathrm{x}, \mathrm{y}_c)$

- The loss metric is additively decomposed over each y_i variables, $\mathrm{loss}(\hat{\boldsymbol{y}}, \check{\boldsymbol{y}}) = \sum_{i=1}^{n} \mathrm{loss}(\hat{y}_i, \check{y}_i)$

- Focus on pairwise graphical models: interactions between label = edges in graphs

Dual:

$$\min_{\theta_e, \theta_v} \mathbb{E}_{\mathbf{X},\mathbf{Y}\sim\tilde{P}} \max_{\check{P}(\check{\mathbf{y}}|\mathbf{x})} \min_{\hat{P}(\hat{\mathbf{y}}|\mathbf{x})} \sum_{\hat{\mathbf{y}}, \check{\mathbf{y}}} \hat{P}(\hat{\mathbf{y}}|\mathbf{x}) \check{P}(\check{\mathbf{y}}|\mathbf{x}) \Big[\sum_i^n \mathrm{loss}(\hat{y}_i, \check{y}_i)$$
$$+ \theta_e \cdot \sum_{(i,j)\in E} [\phi(\mathbf{x}, \check{y}_i, \check{y}_j) - \phi(\mathbf{x}, y_i, y_j)] + \theta_v \cdot \sum_i^n [\phi(\mathbf{x}, \check{y}_i) - \phi(\mathbf{x}, y_i)] \Big]$$

θ_e: Lagrange multipliers for constraints with edge features
θ_v: Lagrange multipliers for constraints with node features

AGM | Marginal Formulation

Dual:

$$\min_{\theta_e,\theta_v} \mathbb{E}_{\mathbf{X},\mathbf{Y}\sim\tilde{P}} \max_{\check{P}(\check{\mathbf{y}}|\mathbf{x})} \min_{\hat{P}(\hat{\mathbf{y}}|\mathbf{x})} \sum_{\hat{\mathbf{y}},\check{\mathbf{y}}} \hat{P}(\hat{\mathbf{y}}|\mathbf{x})\check{P}(\check{\mathbf{y}}|\mathbf{x})\Big[\sum_i^n \mathrm{loss}(\hat{y}_i,\check{y}_i)$$
$$+\,\theta_e\cdot\sum_{(i,j)\in E}[\phi(\mathbf{x},\check{y}_i,\check{y}_j)-\phi(\mathbf{x},y_i,y_j)]+\theta_v\cdot\sum_i^n[\phi(\mathbf{x},\check{y}_i)-\phi(\mathbf{x},y_i)]\Big]$$

Dual | Marginal Formulation:

$$\min_{\theta_e,\theta_v} \mathbb{E}_{\mathbf{X},\mathbf{Y}\sim\tilde{P}} \max_{\check{P}(\check{\mathbf{y}}|\mathbf{x})} \min_{\hat{P}(\hat{\mathbf{y}}|\mathbf{x})} \Big[\sum_i^n \sum_{\hat{y}_i,\check{y}_i} \hat{P}(\hat{y}_i|\mathbf{x})\check{P}(\check{y}_i|\mathbf{x}) loss(\hat{y}_i,\check{y}_i)$$
$$+\sum_{(i,j)\in E}\sum_{\check{y}_i,\check{y}_j}\check{P}(\check{y}_i,\check{y}_j|\mathbf{x})\,[\theta_e\cdot\phi(\mathbf{x},\check{y}_i,\check{y}_j)]-\sum_{(i,j)\in E}\theta_e\cdot\phi(\mathbf{x},y_i,y_j)$$
$$+\sum_i^n\sum_{\check{y}_i}\check{P}(\check{y}_i|\mathbf{x})\,[\theta_v\cdot\phi(\mathbf{x},\check{y}_i)]-\sum_i^n\theta_v\cdot\phi(\mathbf{x},y_i)\Big],$$

Similar to CRF and SSVM:
General Graphical Models:
Intractable

Focus:
Graphs with low tree-width, e.g.: chain, tree.
Tractable optimization

Predictor's probability $\hat{P}(\hat{\boldsymbol{y}}|\boldsymbol{x})$ can be decomposed into node marginal probability $\hat{P}(\hat{y}_i|x)$
Adversary's probability $\check{P}(\check{\boldsymbol{y}}|\boldsymbol{x})$ can be decomposed into node and edge marginal probability $\check{P}(\check{y}_i|x)$ and $\check{P}(\check{y}_i,\check{y}_j|x)$

AGM | Optimization

Matrix Notation (Tree Structure AGM):

$$\min_{\theta_e,\theta_v} \mathbb{E}_{\mathbf{X},\mathbf{Y}\sim\tilde{P}} \max_{\mathbf{Q}} \min_{\mathbf{p}} \sum_i^n \Big[\mathbf{p}_i \mathbf{L}_i (\mathbf{Q}_{pt(i);i}^{\mathrm{T}} \mathbf{1}) + \Big\langle \mathbf{Q}_{pt(i);i} - \mathbf{Z}_{pt(i);i}, \textstyle\sum_l \theta_e^{(l)} \mathbf{W}_{pt(i);i;l} \Big\rangle$$
$$+ (\mathbf{Q}_{pt(i);i}^{\mathrm{T}} \mathbf{1} - \mathbf{z}_i)^{\mathrm{T}} (\textstyle\sum_l \theta_v^{(l)} \mathbf{w}_{i;l}) \Big]$$
$$\text{subject to: } \mathbf{Q}_{pt(pt(i));pt(i)}^{\mathrm{T}} \mathbf{1} = \mathbf{Q}_{pt(i);i} \mathbf{1}, \; \forall i \in \{1, \ldots, n\},$$

Optimization Techniques:

- Stochastic (sub)-gradient descent
  (outer optimization for θ_e and θ_v)
- Dual decomposition (inner $\boldsymbol{Q}$ optimization)
- Discrete optimal transport solver (recovering $\boldsymbol{Q}$)
- Closed-form solution (inner $\boldsymbol{p}$ optimization)

Runtime (for a single subgradient update):

- Depends on the loss metric used
- For the additive zero-one loss metric (Hamming loss)

$O(nlk \log k + nk^2)$

k: # classes, n: # nodes,

l: # iterations in dual decomposition

CRF	SSVM
$O(nk^2)$	$O(nk^2)$

General graphs low tree-width

$O\big(nlwk^{(w+1)} \log k + nk^{2(w+1)}\big)$

n: # cliques, w: treewidth of the graph

AGM | Consistency

If the loss function is additive

AGM is consistent

when f is optimized over all measurable functions on the input space

AGM is also consistent

when f is optimized over a restricted set of functions:

all measurable function that are additive over the edge and node potentials.

AGM | Experiments (1)

Facial Emotion Intensity Prediction (Chain Structure, Labels with Ordinal Category)

- Each node: 3 class classification: *neutral = 1< increasing = 2 < apex = 3*
- 167 sequences
- Ordinal loss metrics: zero-one loss, absolute loss, and squared loss
- Weighted and unweighted. Weights reflect the focus of prediction (e.g. focus more on latest nodes)

Results:

Table 1: The average loss metrics for the emotion intensity prediction. Bold numbers indicate the best or not significantly worse than the best results (paired t-test with $\alpha = 0.05$).

Loss metrics	AGM	CRF	SSVM
zero-one, unweighted	0.34	**0.32**	0.37
absolute, unweighted	**0.33**	0.34	0.40
quadratic, unweighted	**0.38**	**0.38**	0.40
zero-one, weighted	**0.28**	0.32	0.29
absolute, weighted	**0.29**	0.36	**0.29**
quadratic, weighted	0.36	0.40	**0.33**
average	0.33	0.35	0.35
# bold	4	2	2

AGM | Experiments (2)

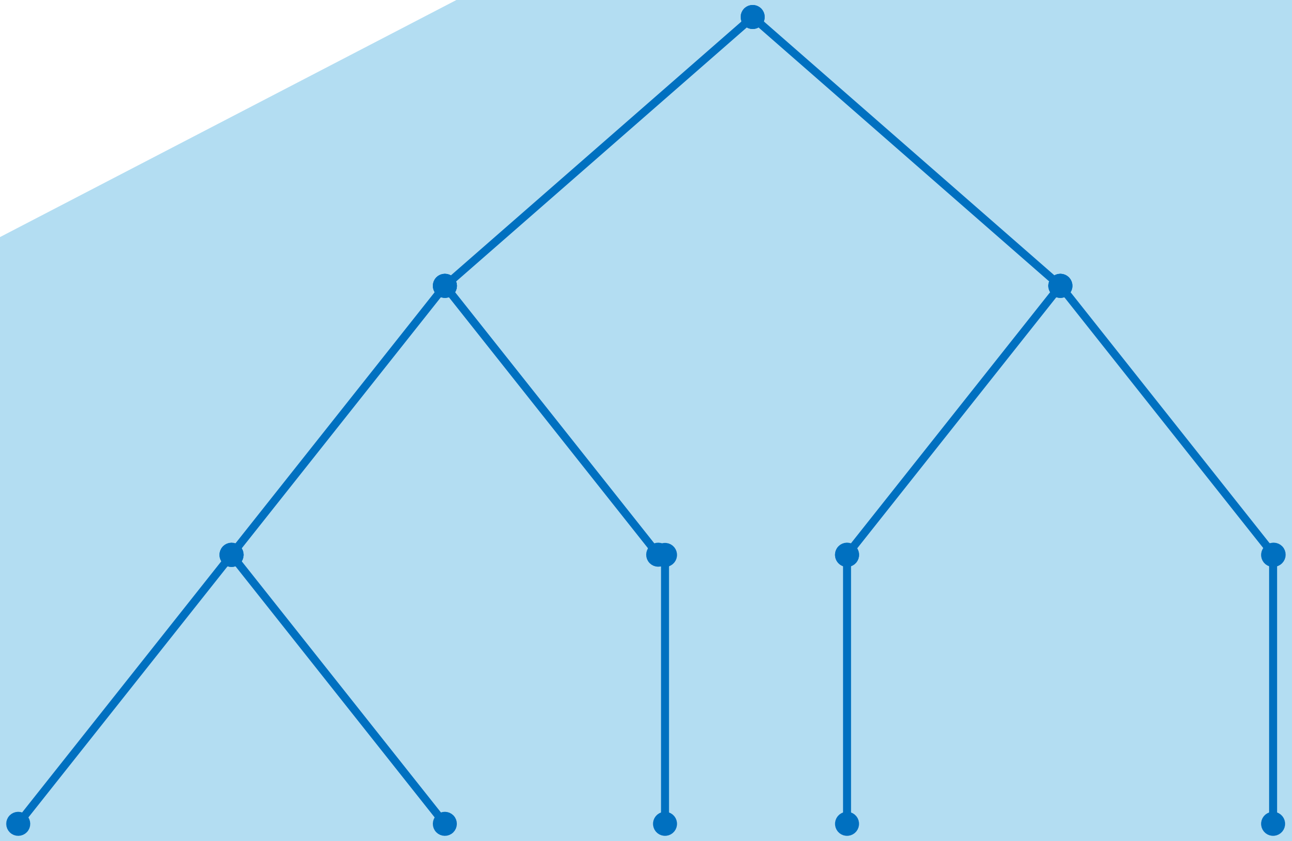
Semantic Role Labeling (Tree Structure)

- Predict label of each node given known parse tree.
- Cost-sensitive loss metric is used reflect the importance of each label
- CoNLL 2005 dataset

Results:

Table 2: The average loss metrics for the semantic role labeling task.

Loss metrics	AGM	CRF	SSVM
cost-sensitive loss	0.14	0.19	0.14



Conditional Graphical Models

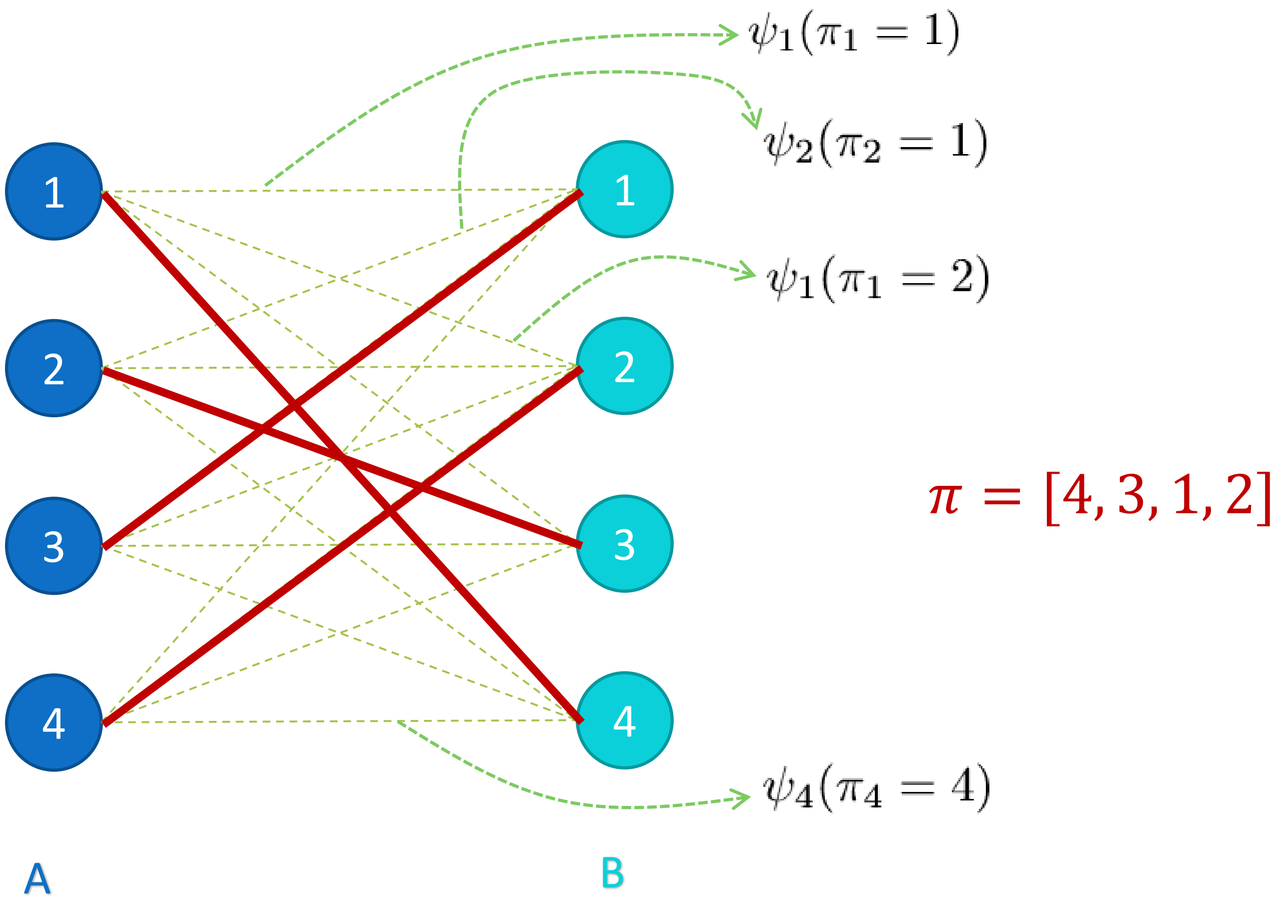
	Performance-Aligned?	Consistent?
Conditional Random Field (CRF) (Lafferty et. al., 2001)	✗	✓
Structured SVM (Tsochantaridis et. al., 2005)	✓	✗
Adversarial Graphical Models (our approach)	✓	✓

Bipartite Matching in Graphs

Based on:

Rizal Fathony*, Sima Behpour*, Xinhua Zhang, Brian D. Ziebart. "*Efficient and Consistent Adversarial Bipartite Matching*". International Conference on Machine Learning (ICML), 2018.

Bipartite Matching Task



Maximum weighted bipartite matching:

$$\max_{\pi \in \Pi} \psi(\pi) = \max_{\pi \in \Pi} \sum_i \psi_i(\pi_i)$$

Machine learning task:

Learn the appropriate weights $\psi_i(\cdot)$

Objective:

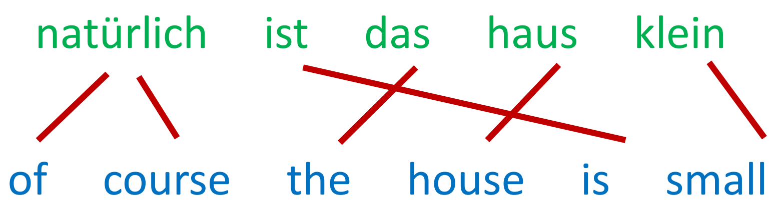
Minimize a loss metric, e.g., the Hamming loss

$$\text{loss}_{\text{Ham}}(\pi, \pi') = \sum_{i=1}^{n} 1(\pi'_i \neq \pi_i)$$

Learning Bipartite Matching | Applications

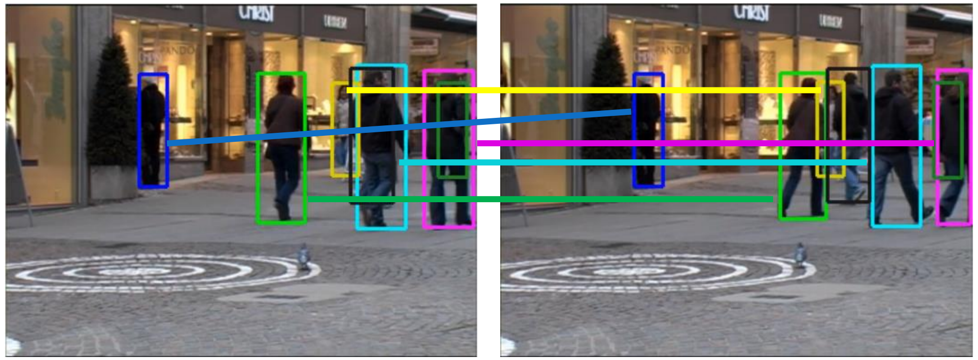
❶ Word alignment

(Taskar et. al., 2005; Pado & Lapta, 2006; Mac-Cartney et. al., 2008)



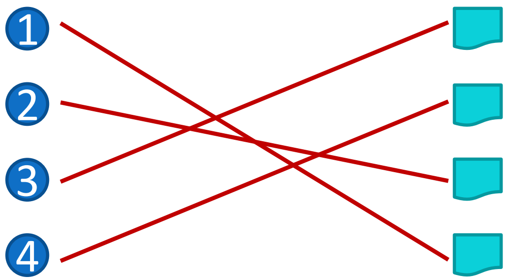
❷ Correspondence between images

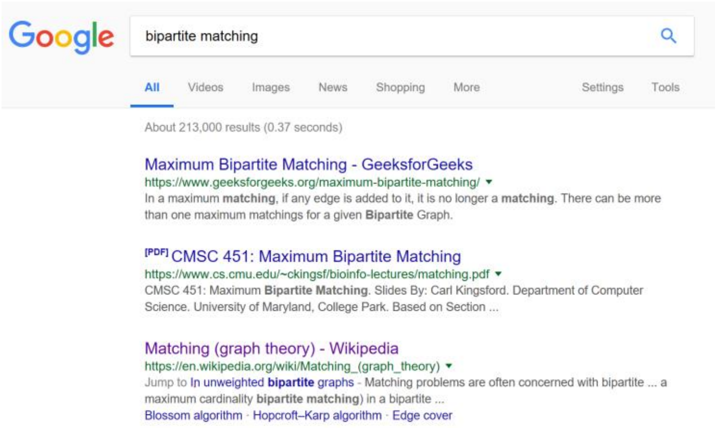
(Belongie et. al., 2002; Dellaert et. al., 2003)



❸ Learning to rank documents

(Dwork et. al., 2001; Le & Smola, 2007)





A non-bipartite matching task can be converted to a bipartite matching problem

Previous Approaches for Bipartite Matching

1 CRF (Petterson et. al., 2009; Volkovs & Zemel, 2012)

$$P_\psi(\pi) = \frac{1}{Z_\psi} \exp\left(\sum_{i=1}^{n} \psi_i(\pi_i)\right)$$

$$Z_\psi = \sum_{\pi} \prod_{i=1}^{n} \exp\left(\psi_i(\pi_i)\right) = \mathrm{perm}(\mathbf{M})$$

$$\text{where} M_{i,j} = \exp\left(\psi_i(j)\right)$$

✓ **Fisher Consistent**
Produce Bayes optimal prediction in ideal case

✗ **Computationally intractable**
Normalization term requires matrix permanent computation (a #P-hard problem).
An approximation is needed.

2 Structured SVM (Tsochantaridis et. al., 2005)

$$\min_{\psi} \mathbb{E}_{\pi \sim \tilde{P}} \left[\underbrace{\max_{\pi'} \left\{\mathrm{loss}(\pi, \pi') + \psi(\pi')\right\} - \psi(\pi)}^{\text{solved using constraint generation}}\right]$$

$\tilde{P}$ is the empirical distribution

✓ **Computationally Efficient**
Hungarian algorithm for computing the maximum violated constraints

✗ **No Fisher consistency guarantee**
Based on Multiclass SVM-CS
Not consistent for distribution with no majority label

Adversarial Bipartite Matching (our approach)

Primal:

$$\min_{\hat{P}(\hat{\pi}|x)} \max_{\check{P}(\check{\pi}|x)} \mathbb{E}_{x\sim\tilde{P};\hat{\pi}|x\sim\hat{P};\check{\pi}|x\sim\check{P}} [\text{loss}(\hat{\pi},\check{\pi})]$$

$$\text{s.t. } \mathbb{E}_{x\sim\tilde{P};\check{\pi}|x\sim\check{P}}\left[\sum_{i=1}^{n} \phi_i(x,\check{\pi}_i)\right] = \mathbb{E}_{(x,\pi)\sim\tilde{P}}\left[\sum_{i=1}^{n} \phi_i(x,\pi_i)\right]$$

Predictor Adversary

Augmented Hamming loss matrix for $n = 3$ permutations

	$\check{\pi}=123$	$\check{\pi}=132$	$\check{\pi}=213$	$\check{\pi}=231$	$\check{\pi}=312$	$\check{\pi}=321$
$\hat{\pi}=123$	$0+\delta_{123}$	$2+\delta_{132}$	$2+\delta_{213}$	$3+\delta_{231}$	$3+\delta_{312}$	$2+\delta_{321}$
$\hat{\pi}=132$	$2+\delta_{123}$	$0+\delta_{132}$	$3+\delta_{213}$	$2+\delta_{231}$	$2+\delta_{312}$	$3+\delta_{321}$
$\hat{\pi}=213$	$2+\delta_{123}$	$3+\delta_{132}$	$0+\delta_{213}$	$2+\delta_{231}$	$2+\delta_{312}$	$3+\delta_{321}$
$\hat{\pi}=231$	$3+\delta_{123}$	$2+\delta_{132}$	$2+\delta_{213}$	$0+\delta_{231}$	$3+\delta_{312}$	$2+\delta_{321}$
$\hat{\pi}=312$	$3+\delta_{123}$	$2+\delta_{132}$	$2+\delta_{213}$	$3+\delta_{231}$	$0+\delta_{312}$	$2+\delta_{321}$
$\hat{\pi}=321$	$2+\delta_{123}$	$3+\delta_{132}$	$3+\delta_{213}$	$2+\delta_{231}$	$2+\delta_{312}$	$0+\delta_{321}$

size:

$n! \times n!$

<u>Intractable</u>

for modestly-sized n

Dual:

$$\min_{\theta} \mathbb{E}_{x,\pi\sim\tilde{P}} \min_{\hat{P}(\hat{\pi}|x)} \max_{\check{P}(\check{\pi}|x)} \mathbb{E}_{\substack{\hat{\pi}|x\sim\hat{P}\\ \check{\pi}|x\sim\check{P}}} \left[\text{loss}(\hat{\pi},\check{\pi}) + \theta\cdot\sum_{i=1}^{n}\left(\phi_i(x,\check{\pi}_i) - \phi_i(x,\pi_i)\right)\right]$$

Hamming loss Lagrangian term δ

Polytope of the Permutation Mixtures

Dual:

$$\min_{\theta} \mathbb{E}_{(x,\pi)\sim\tilde{P}} \min_{\hat{P}(\hat{\pi}|x)} \max_{\check{P}(\check{\pi}|x)} \mathbb{E}_{\hat{\pi}|x\sim\hat{P};\check{\pi}|x\sim\check{P}} \left[\sum_{i=1}^{n} I(\pi'_i \neq \pi_i) + \theta \cdot \sum_{i=1}^{n} (\phi_i(x,\check{\pi}_i) - \phi_i(x,\pi_i)) \right]$$

Marginal Distribution Matrices:

Predictor

$\mathbf{P} =$

	1	2	3
$\hat{\pi}_1$	$p_{1,1}$	$p_{1,2}$	$p_{1,3}$
$\hat{\pi}_2$	$p_{2,1}$	$p_{2,2}$	$p_{2,3}$
$\hat{\pi}_3$	$p_{3,1}$	$p_{3,2}$	$p_{3,3}$

$p_{i,j} = \hat{P}(\hat{\pi}_i = j)$

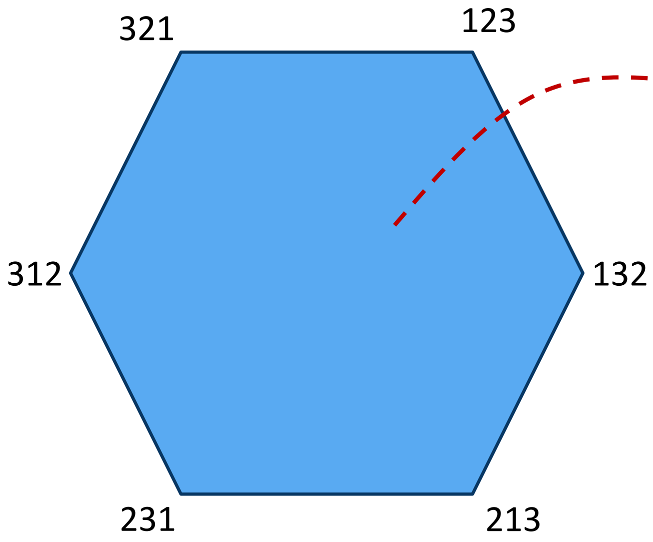
Adversary

$\mathbf{Q} =$

	1	2	3
$\check{\pi}_1$	$q_{1,1}$	$q_{1,2}$	$q_{1,3}$
$\check{\pi}_2$	$q_{2,1}$	$q_{2,2}$	$q_{2,3}$
$\check{\pi}_3$	$q_{3,1}$	$q_{3,2}$	$q_{3,3}$

$q_{i,j} = \check{P}\,(\check{\pi}_i = j)$

Birkhoff – Von Neumann theorem:



convex polytope whose points are doubly stochastic matrix

$$\mathbf{P1} = \mathbf{P}^{\top}\mathbf{1} = \mathbf{Q1} = \mathbf{Q}^{\top}\mathbf{1} = \mathbf{1}$$

reduce the space of optimization: from $O(n!)$ to $O(n^2)$

Marginal Distribution Formulation

Dual:

$$\min_{\theta} \mathbb{E}_{(x,\pi)\sim\tilde{P}} \min_{\hat{P}(\hat{\pi}|x)} \max_{\check{P}(\check{\pi}|x)} \mathbb{E}_{\hat{\pi}|x\sim\hat{P};\check{\pi}|x\sim\check{P}} \left[\sum_{i=1}^{n} I(\pi'_i \neq \pi_i) + \theta \cdot \sum_{i=1}^{n} (\phi_i(x,\check{\pi}_i) - \phi_i(x,\pi_i)) \right]$$

Marginal Formulation:

Rearrange the optimization order and add regularization and smoothing penalties

$$\max_{\mathbf{Q}\geq\mathbf{0}} \min_{\theta} \frac{1}{m} \sum_{i=1}^{m} \min_{\mathbf{P}_i\geq\mathbf{0}} \left[\langle \mathbf{Q}_i - \mathbf{Y}_i, \textstyle\sum_k \theta_k \mathbf{X}_{i,k} \rangle - \langle \mathbf{P}_i, \mathbf{Q}_i \rangle + \frac{\mu}{2}\|\mathbf{P}_i\|_F^2 - \frac{\mu}{2}\|\mathbf{Q}_i\|_F^2 \right] + \frac{\lambda}{2}\|\theta\|_2^2$$

$$\text{s.t.}: \mathbf{P}_i\mathbf{1} = \mathbf{P}_i^\top\mathbf{1} = \mathbf{Q}_i\mathbf{1} = \mathbf{Q}_i^\top\mathbf{1} = \mathbf{1}, \quad \forall i$$

Optimization Techniques Used:

- Outer (Q) : projected Quasi-Newton (Schmidt, et.al., 2009)
- Inner (θ) : closed-form solution
- Inner (P) : projection to doubly-stochastic matrix
- Projection to doubly-stochastic matrix : ADMM

Consistency

Empirical Risk Perspective of Adversarial Bipartite Matching

$$\min_{\theta} \mathbb{E}_{\substack{x\sim P \\ \pi|x\sim\tilde{P}}} \left[AL^{\text{perm}}_{f_\theta}(x,\pi) \right]$$

$$\text{where: } AL^{\text{perm}}_{f_\theta}(x,\pi) \triangleq \min_{\hat{P}(\hat{\pi}|x)} \max_{\check{P}(\check{\pi}|x)} \mathbb{E}_{\substack{\hat{\pi}|x\sim\hat{P} \\ \check{\pi}|x\sim\check{P}}} \left[\text{loss}(\hat{\pi},\check{\pi}) + f_\theta(x,\check{\pi}) - f_\theta(x,\pi) \right]$$

AL^{perm} is consistent

when f is optimized over all measurable functions on the input space (x, π)

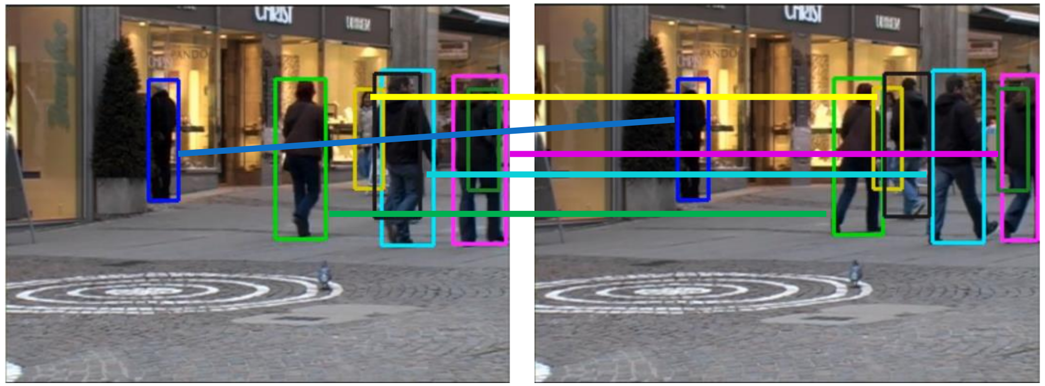
AL^{perm} is also consistent

f is optimized over a restricted set of functions: $f(x,\pi) = \sum_i g_i(x, \pi_i)$

when g is allowed to be optimized over all measurable functions on the individual input space (x, π_i)

Experiments

Application: Video Tracking



Public Benchmark Datasets

Table 3. Dataset properties

DATASET	# ELEMENTS	# EXAMPLES
TUD-CAMPUS	12	70
TUD-STADTMITTE	16	178
ETH-SUNNYDAY	18	353
ETH-BAHNHOF	34	999
ETH-PEDCROSS2	30	836

Empirical runtime (until convergence)

Table 5. Running time (in seconds) of the model for various number of elements n with fixed number of samples ($m = 50$)

DATASET	# ELEMENTS		ADV MARG.		SSVM
CAMPUS	12	1.0	1.96	1.0	0.22
STADTMITTE	16	1.3	2.46	1.2	0.25
SUNNYDAY	18	1.5	2.75	1.4	0.15
PEDCROSS2	30	2.5	8.18	4.2	0.26
BAHNHOF	34	2.8	9.79	5.0	0.31

relative: 12=1.0 relative: 1.96=1.0

Adversarial. Marginal Formulation:
grows (roughly) quadratically in n

CRF: impractical even for $n = 20$
(Petterson et. al., 2009)

Experiment Results

Table 1: The mean and standard deviation (in parenthesis) of the average accuracy (1 - the average Hamming loss) for the adversarial bipartite matching model compared with Structured-SVM.

Training/ Testing	Adv. Bipartite Matching	Structured SVM
Campus/ Stadtmitte	0.662 (0.08)	0.662 (0.08)
Stadtmitte/ Campus	0.667 (0.11)	0.660 (0.12)
Bahnhof/ Sunnyday	**0.754** (0.10)	0.729 (0.15)
Pedcross2/ Sunnyday	**0.750** (0.10)	0.736 (0.13)
Sunnyday/ Bahnhof	**0.751** (0.18)	0.739 (0.20)
Pedcross2/ Bahnhof	**0.763** (0.16)	0.731 (0.21)
Bahnhof/ Pedcross2	**0.714** (0.16)	0.701 (0.18)
Sunnyday/ Pedcross2	**0.712** (0.17)	0.700 (0.18)

6 pairs of dataset significantly outperforms SSVM

2 pairs of dataset competitive with SSVM

Bipartite Matching in Graphs

	Efficient?	Consistent?	Perform well?
Conditional Random Field (CRF) (Petterson et. al., 2009; Volkovs & Zemel, 2012)	✗	✓	?
Structured SVM (Tsochantaridis et. al., 2005)	✓	✗	–
Adversarial Bipartite Matching (our approach)	✓	✓	✓

Conclusion

Robust Adversarial Learning Algorithms

- ✓ Align better with the loss/performance metric
  (by incorporating the metric into its learning objective)
- ✓ Provide Fisher consistency guarantee
- ✓ Computationally efficient
- ✓ Perform well in practice

Ongoing and Future Works

Ongoing and Future Works (1)

1. Fairness and Privacy in Machine Learning

Important issues in automated decision using ML algorithms.

Require the algorithm to produce fair prediction / privacy-preserving prediction.

Our formulation only enforces constraints on the adversary.

$$\min_{\hat{P}(\hat{Y}|\mathbf{X})} \max_{\check{P}(\check{Y}|\mathbf{X})} \mathbb{E}_{\mathbf{X},Y\sim\tilde{P};\hat{Y}|\mathbf{X}\sim\hat{P};\check{Y}|\mathbf{X}\sim\check{P}}\left[\text{loss}(\hat{Y},\check{Y})\right]$$

$$\text{s.t. } \mathbb{E}_{\mathbf{X}\sim\tilde{P};\check{Y}|\mathbf{X}\sim\check{P}}[\phi(\mathbf{X},\check{Y})] = \mathbb{E}_{\mathbf{X},Y\sim\tilde{P}}[\phi(\mathbf{X},Y)]$$

Add fairness / privacy constraints to the predictor?

2. Multivariate Performance Metrics

Many ML applications uses multivariate performance metrics to evaluate the prediction.

- F_β-score
- Precision/Recall @k
- Area under ROC curve (AOC)

How will the optimization techniques change to accommodate these metrics?

What if we have both structure in the label inter-actions as well as structure in the loss metrics?

e.g. Bipartite Matching with F1-score

Ongoing and Future Works (2)

3. Structured Prediction & Graphical Models

More complex graphical structures are popular in some applications, e.g. computer vision.

Exact learning algorithms for AGM in this case may be intractable.

Can we develop learning algorithms for general graphical models?

What kind of approximation algorithms can be applicable?

4. Deep Learning

Deep learning has been successfully applied to many prediction problems.

Most of deep learning architectures are not designed to optimize customized loss metrics.

How can the robust adversarial learning approach help designing deep learning architectures?

Ongoing and Future Works (3)

5. Multitask Learning

In some problems, learning multiple tasks with different metrics simultaneously is desirable.

What if we want to optimize multiple different loss metrics simultaneously?

How will it change the optimization?

6. Statistical Theory of Loss Functions

In multiclass classification problem, both $AL^{0\text{-}1}$ and SVM-LLW are Fisher consistent. However, their performances are quite different.

Is there any stronger statistical guarantee that can separate high-performing Fisher consistent algorithm from the low-performing ones?

Collaborators

AL
Anqi Liu

KA
Kaiser Asif

BZ
Prof. Brian Ziebart

MB
Mohammad Bashiri

SB
Sima Behpour

XZ
Prof. Xinhua Zhang

AR
Ashkan Rezaei

Publications

- **Consistent Robust Adversarial Prediction for General Multiclass Classification**
  Rizal Fathony, Kaiser Asif, Anqi Liu, Mohammad Bashiri, Xinhua Zhang, Brian D. Ziebart.
  JMLR submission in preparation.

- **Distributionally Robust Graphical Models**
  Rizal Fathony, Ashkan Rezaei, Mohammad Bashiri, Xinhua Zhang, Brian D. Ziebart.
  Advances in Neural Information Processing Systems 31 (NIPS), 2018.

- **Efficient and Consistent Adversarial Bipartite Matching**
  Rizal Fathony*, Sima Behpour*, Xinhua Zhang, Brian D. Ziebart.
  International Conference on Machine Learning (ICML), 2018.

- **Adversarial Surrogate Losses for Ordinal Regression**
  Rizal Fathony, Mohammad Bashiri, Brian D. Ziebart.
  Advances in Neural Information Processing Systems 30 (NIPS), 2017.

- **Adversarial Multiclass Classification: A Risk Minimization Perspective**
  Rizal Fathony, Anqi Liu, Kaiser Asif, Brian D. Ziebart.
  Advances in Neural Information Processing Systems 29 (NIPS), 2016.

- **Kernel Robust Bias-Aware Prediction under Covariate Shift**
  Anqi Liu, **Rizal Fathony**, Brian D. Ziebart. ArXiv Preprints, 2016.

Thank You