Multiplicative Filter Networks

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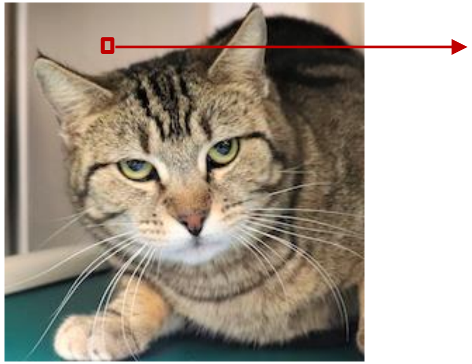
Task

Approximating Low-dimensional-but-complex Functions

- Learn a neural representation of low-dimensional-but-complex functions
- Example: Image Representation



Input:

Pixel coordinate ($\mathbb{R}^2$)

(x, y)

Output:

RGB values ($\mathbb{R}^3$)

(r, g, b)

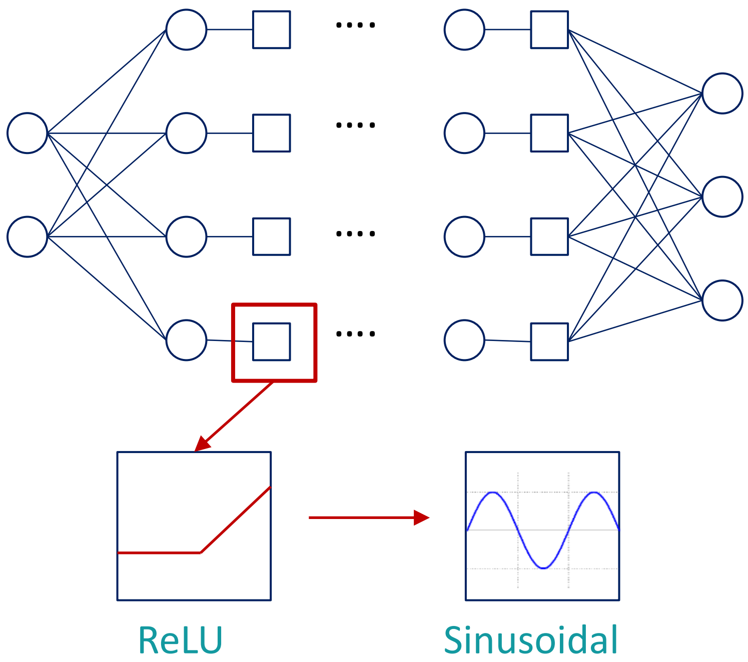
- Other Examples:
  - Image Generalization
  - Poisson Image Reconstruction from Gradients and Laplacian
  - Solving Wave Function
  - Shape Representation
  - 3D Inverse Rendering
  - …

Previous Works

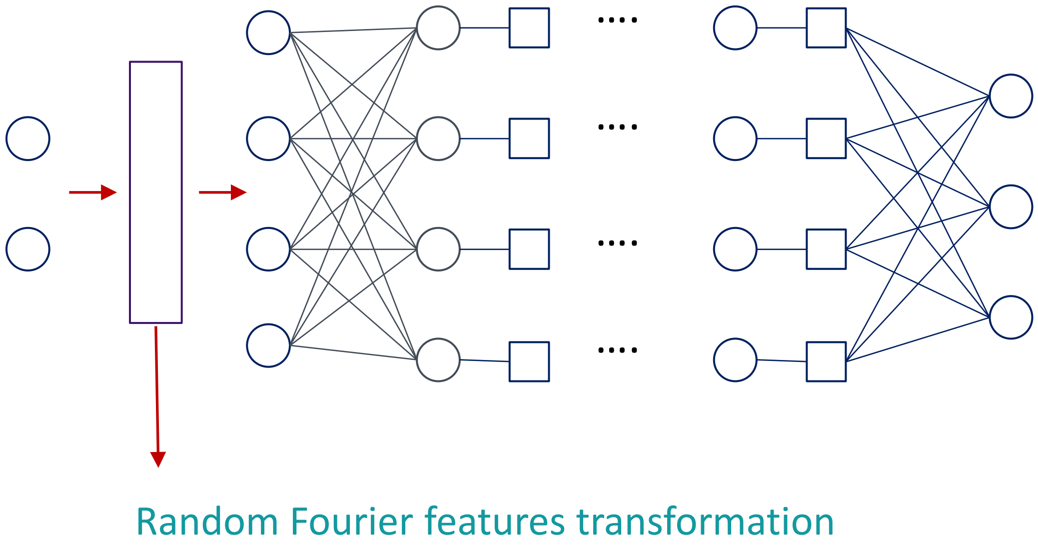
Sinusoidal Activation & Fourier Features

- Standard ReLU networks performs miserably on the task (Sitzmann et.al., 2020)

SIREN (Sitzmann, et.al, 2020)



Fourier features network
(Tancik, et.al, 2020)



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Previous Works

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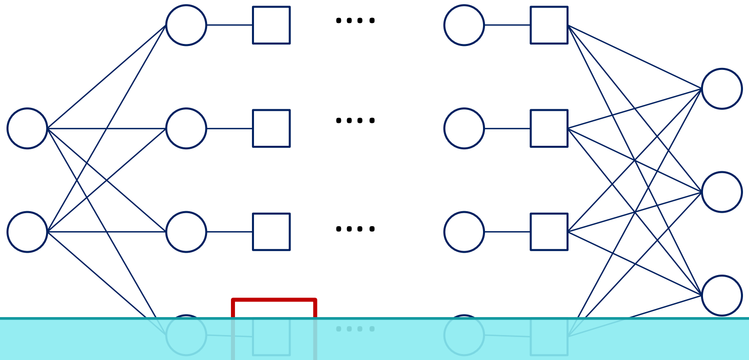
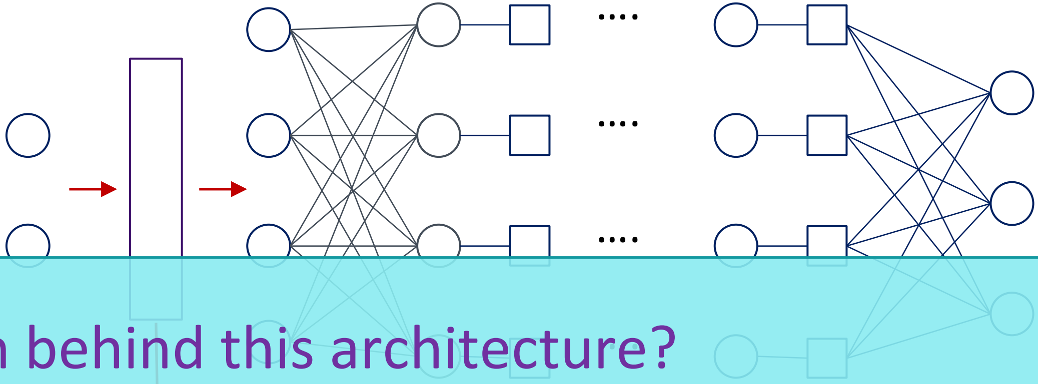


Fourier features network (Tancik, et.al, 2020)



Problem: What is the intuition behind this architecture?

Periodic activations → something akin to a Fourier transform,

Network's compositional structure → difficult to analyze theoretically

Our Model
Multiplicative Filter Networks

- **Standard Compositional Networks**

Recurrence Formulation:

$$z^{(1)} = x$$
$$z^{(i+1)} = \sigma\left(W^{(i)} z^{(i)} + b^{(i)}\right),\ i = 1, \ldots, k-1$$
$$f(x) = W^{(k)} z^{(k)} + b^{(k)}$$

σ : non-linear activation function

- **Multiplicative Filter Networks (MFNs)**

Recurrence Formulation:

$$z^{(1)} = g\left(x; \theta^{(1)}\right)$$
$$z^{(i+1)} = \left(W^{(i)} z^{(i)} + b^{(i)}\right) \circ g\left(x; \theta^{(i+1)}\right),\ i = 1, \ldots, k-1$$
$$f(x) = W^{(k)} z^{(k)} + b^{(k)}$$

$\circ$: elementwise multiplication, g : non-linear function

1) FourierNet: sinusoidal filter

$$g(x; \theta^{(i)}) = \sin(\omega^{(i)} x + \phi^{(i)})$$

2) GaborNet: Gabor filter

$$g_j(x; \theta^{(i)}) = \exp\left(-\frac{\gamma_j^{(i)}}{2}\left\|x - \mu_j^{(i)}\right\|_2^2\right) \sin\left(\omega_j^{(i)} x + \phi_j^{(i)}\right)$$

Theoretical Benefit

Characterization of the entire function of the networks

- Simple characterization of the entire function of a MFN:

  a linear combination of (an exponential number of) Fourier or Gabor basis functions on the input.

- Two instantiations of MFNs:

  - FourierNet: $$f_j(x) = \sum_{t=1}^{T} \bar{\alpha}_t \sin(\bar{\omega}_t x + \bar{\phi}_t) + \bar{b}$$

  - GaborNet: $$f_j(x) = \sum_{t=1}^{T} \bar{\alpha}_t \exp\left(-\frac{1}{2}\bar{\gamma}_t \|x - \bar{\mu}_t\|^2\right) \sin(\bar{\omega}_t x + \bar{\phi}_t) + \bar{b}$$

for coefficients $\bar{\alpha}_{1:T}$, scales $\bar{\gamma}_{1:T}$, means $\bar{\mu}_{1:T}$, frequencies $\bar{\omega}_{1:T}$, phase offsets $\bar{\phi}_{1:T}$, and bias term $\bar{b}$.

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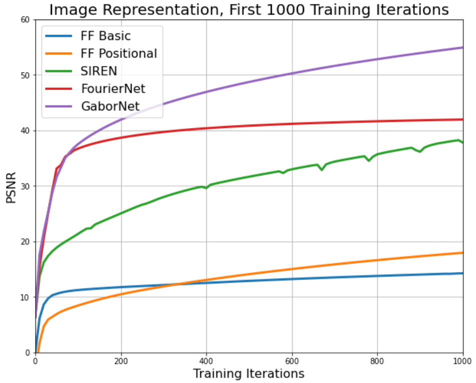
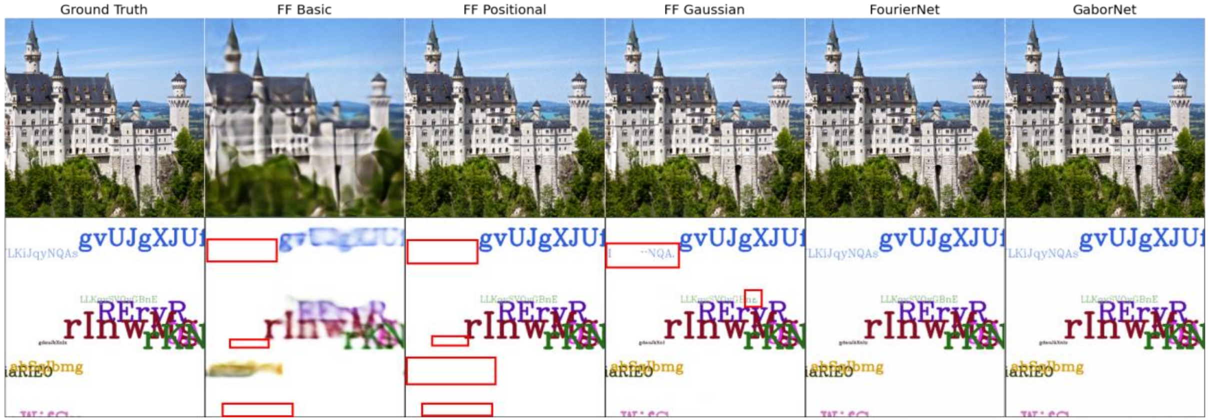
Empirical Benefits
Experiments (1)

- Image & Video Representation



Method	PSNR (in dB)	
	Image	Video
FF Basic	20.13	24.09 ± 1.03
FF Positional	40.09	27.90 ± 0.99
SIREN	56.54	**30.58** ± **0.93**
FOURIERNET	43.32	27.93 ± 0.91
GABORNET	**73.98**	29.83 ± 0.71

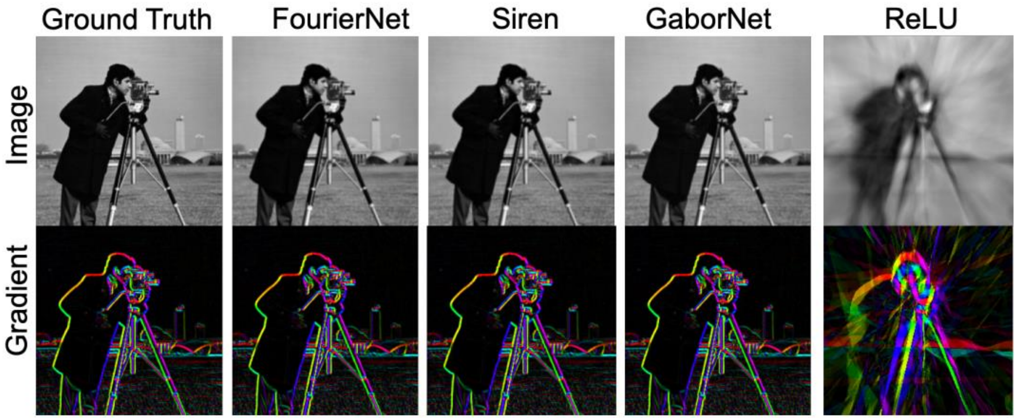
- Image Generalization



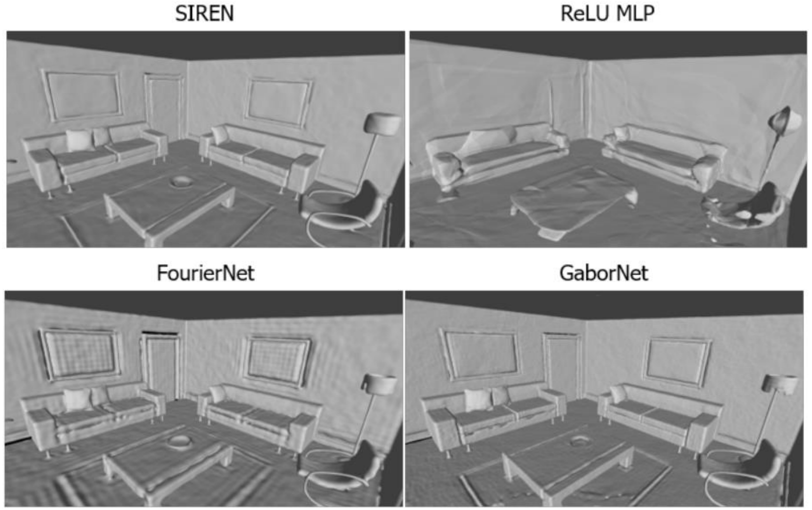
Method	Natural	Text
FF Basic	21.61 ± 2.62	20.50 ± 2.13
FF Positional	25.13 ± 4.01	26.49 ± 3.11
FF Gaussian	25.57 ± 4.18	30.46 ± 1.97
FOURIERNET	26.03 ± 2.77	31.02 ± 2.04
GABORNET	**26.18** ± **2.95**	**31.19** ± **2.00**

Empirical Benefits
Experiments (2)

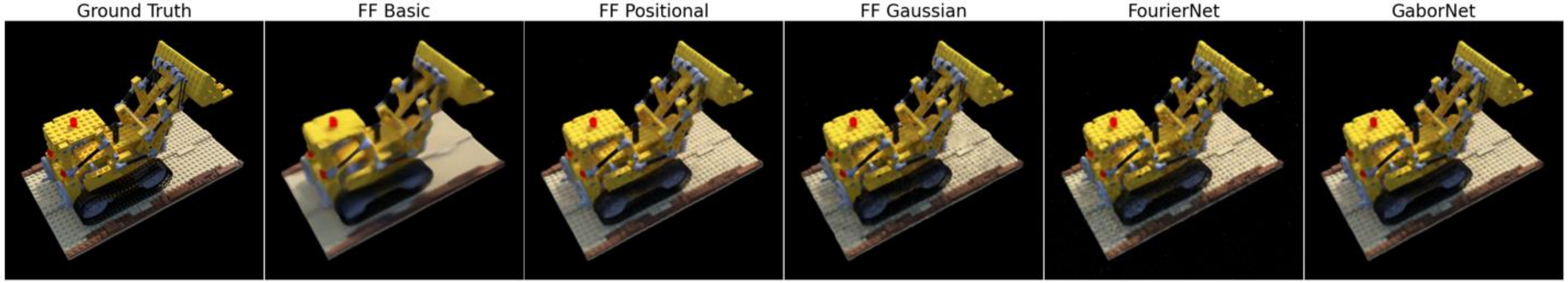
- Poisson Image Reconstruction from Gradients



- Shape Representation via SDF



- 3D Inverse Rendering



Method	PSNR
FF Basic	23.37 ± 0.96
FF Positional	25.76 ± 0.79
FF Gaussian	25.76 ± 0.92
FOURIERNET	25.20 ± 0.72
GABORNET	$\mathbf{25.81 \pm 0.76}$

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Conclusion

- We introduced the Multiplicative Filter Networks (MFNs)
  - Two instantiations:
    1) FourierNet : Sinusoidal Filter
    2) GaborNet : Gabor Filter
  - Avoid composing non-linear function on top of other non-linear functions
- The entire function of the networks
  - Simple
  - Easy to characterize
- Empirical Performance
  - The MFNs perform as well or better than the previous approach
- A standard benchmark for future works

  on modeling low-dimensional-but-complex functions

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Thank You